Damage Spreading and Opinion Dynamics on Scale Free Networks

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We study damage spreading among the opinions of a system of agents, subjected to the dynamics of the Krause-Hegselmann consensus model. The damage consists in a sharp change of the opinion of one or more agents in the initial random opinion configuration, supposedly due to some external factors and/or events. This may help to understand for instance under which conditions special shocking events or targeted propaganda are able to influence the results of elections. For agents lying on the nodes of a Barabási-Albert network, there is a damage spreading transition at a low value ϵ_d of the confidence bound parameter. Interestingly, we find as well that there is some critical value ϵ_s above which the initial perturbation manages to propagate to all other agents.

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Most natural, social and technological systems are continuously subjected to external stimulations of all kinds. Examples are infections, hacker or terrorism attacks, noise, errors, etc. It may happen that, due to these shocks, a sudden change occurs in some feature of a limited number of subjects (thinking about human beings the feature could be health, religion or, like in this paper, opinion), and that successively, through interactions with other members of the system, this perturbation spreads until it eventually affects a big portion of the system. The study of such processes is of great importance, in order to take the effects of eventual future local failures under control, but it is also an important method to investigate the dynamics of a system.

Damage spreading (DS) was originally used by Kauffman [1] as a tool for studying biologically motivated dynamical systems. In physics, the first investigations focused on the Ising model [2]. Here one starts from some arbitrary configuration of spins and creates a replica by flipping one or more spins; after that one lets both configurations evolve towards equilibrium according to the chosen dynamics under the same thermal noise (i.e. identical sequences of random numbers). It turns out that there is a temperature T_d, near the Curie point, which separates a phase where the damage heals from a phase in which the perturbation extends to a finite fraction of the spins of the system.

Meanwhile there is a sizeable literature on this problem, which finds applications in many fields of modern science. Epidemiology, for example, is by definition the study of a particular DS problem [4]. After the recent discovery that many systems in nature and society can be described as complex networks, scale free graphs have been intensively investigated and many classical problems have been reformulated on such special topologies [5]. In particular it is very interesting to understand the mechanisms by which diseases, information, computer viruses, etc. spread over networks.

In this paper the network represents the system of acquaintances between people and we study the following problem: suppose we have a community of voters at the beginning of an electoral campaign, during which the voters shape their own opinions through relationships with their friends. If a small set of voters for any reason suddenly change their mind at this initial stage, would the final outcome of the election be influenced and, if yes, to which extent? Very recent history delivers a dramatic example: the shock caused to the Spanish people by the bombs in Madrid on March the 11th 2004 turned over the outcome of the national elections, which seemed already decided till that day.

What we need is a model that describes how people convince each other. In the last years quite a few models of opinion dynamics have been proposed, like those of Galam [6], Deffuant et al. [7], Krause-Hegselmann (KH) [8], or Sznajd [9], and sociophysics simulations have become a fruitful field of research [10, 11]. As far as our DS problem is concerned, some results on the Sznajd model recently appeared [12], but for an improbable society where agents sit on the sites of a square lattice.

Here we present the first systematic study on the subject. Our agents are on the nodes of a Barabási-Albert (BA) network [13], which represents a more realistic model for the structure of social relationships. The network is constructed by means of a growth process starting from m nodes which are all connected to each other. Nodes are then added one by one and each of them forms m edges with the existing vertices, such that the probability to get linked to a node is proportional to the number of its neighbours. At the end, the number of agents with *degree* k, i.e. having k neighbours, is proportional to $1/k^3$ for k large, independently of m. Our simulations show that the main results are only weakly dependent on m, so we focused on the case $m = 3$.

We adopted the opinion dynamics of the KH consensus model. In this model, the opinions are real numbers between 0 and 1 and a confidence bound parameter ϵ, also real in $[0 : 1]$, is introduced. One starts from a random distribution of opinions. If we want to update the opinion s_i of agent i we have to select among all neighbours of i only those agents whose opinions are compatible with s_i, i.e. those agents j such that $|s_i - s_j| < \epsilon$; next, i takes the average opinion of the compatible agents. We chose to update sequentially the status of the agents, in

an ordered sweep over the whole population; in this way, the dynamics does not require random numbers and is therefore truly deterministic, at variance with other consensus models. At some stage, the system will converge to a configuration that the dynamics is unable to modify. This configuration represents the equilibrium state and the final opinion distribution, which is given by a set of δ-functions, depends on ϵ. On BA networks, we found that the threshold for complete consensus is $\epsilon_c = 1/2$, independently of m.

For the DS analysis we followed the procedure that we exposed above for the Ising model. After creating a random opinion configuration, we produced a replica of it in that we changed the opinion of a single agent i. The results do not depend on the exact number of perturbed sites, as long as they are a fraction of the population N that vanishes in the limit when N goes to infinity [14]. We perturbed the opinion as follows: if $s_i > 1/2$ the new opinion becomes $s_i - 1/2$, otherwise $s_i + 1/2$.

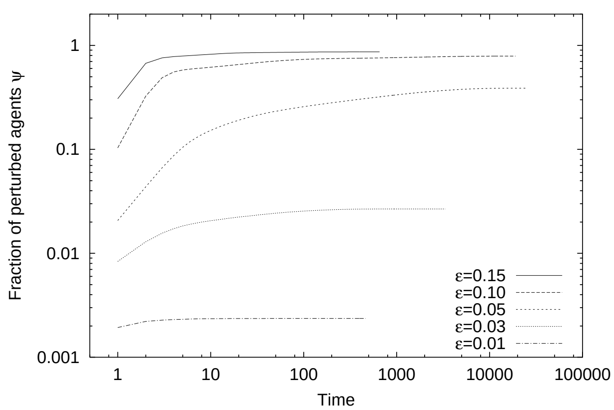

FIG. 1: Time evolution of the fraction of perturbed opinions for several values of ϵ. Here the population is $N = 10000$.

After each sweep over the network we calculated the fraction Ψ of different opinions in the two configurations. The simulation stops when both systems reach their final states, which happens when no agent changed opinion during an iteration. In all our calculations we used double precision real numbers and we decided that two opinions are the same if they differ from each other by less than 10^{-9}, otherwise they are different. For the results to have statistical significance we took averages over 1000 samples for all values of ϵ and of the population N.

We stress that on our network, like in a real society, some nodes are more important than others. If we aim at spreading an opinion in the society, we should better try to convince people with many friends than persons with few social contacts. We then expect that the perturbation on the whole system will be more relevant if we initially damage a hub than a loosely connected node and we investigated both situations.

First, we analyzed the case in which the perturbed node is a hub. Fig. 1 shows the variation with time of Ψ (the time unit is one sweep over the network). There is a characteristic pattern with an initial phase in which Ψ grows rapidly, followed by a very slow relaxation to the final value.

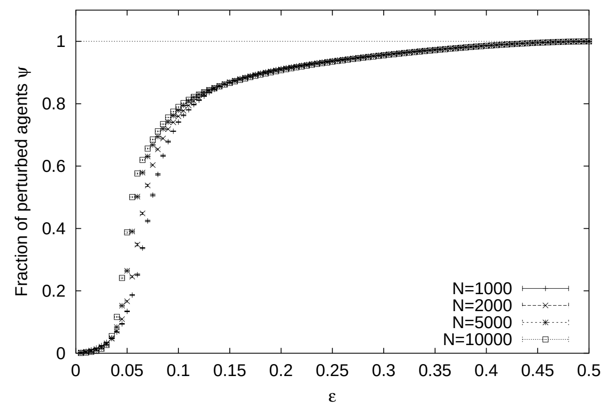

FIG. 2: Variation of Ψ with the confidence bound parameter ϵ.

As we can see from the figure, the relaxation time increases up to $\epsilon \sim 0.1$, then it decreases. At the end of relaxation the number of damaged sites stays constant although the system keeps evolving. This is due to the fact that, at advanced stages of the evolution, the opinions are grouped in clusters. Agents of different clusters cannot interact with each other, as their opinions differ by more than ϵ. Because of that, if a cluster contains perturbed opinions, most of its agents will be sooner or later affected [15], otherwise it can never be reached by the perturbation.

In Fig. 2 we plot Ψ as a function of ϵ, for different network sizes. We see that the damage rises fast with ϵ and that, for ϵ larger than about 0.05, more than half of the agents have been affected. The inflection of the curves at low values of the confidence bound hints to the existence of a DS transition, like in the Ising model. If there is indeed a phase in which the damage affects only a vanishing fraction of the system, we should find that there Ψ goes to zero when N goes to infinity. We then looked for scaling behaviour of Ψ with N. In Fig. 3 we plot Ψ as a function of N for several values of the confidence bound. We can see that the points can be quite well fitted by a simple power law up to $\epsilon \sim 0.015$. For higher values, a saturation to a non-zero Ψ takes place. As estimate of the DS threshold ϵ_d, we took the value which gave the smallest χ^2 for the fit with the power law: we found $\epsilon_d = 0.013(3)$. The error marks the range where the above-mentioned χ^2 (per degree of freedom) is below 1.

Coming back to Fig. 2, we notice that in contrast to the Ising or Kauffman models, we have a second threshold ϵ_s above which all agents are affected by the initial local perturbation. To our knowledge, this is a truly novel feature for a DS process, and we expect it to hold

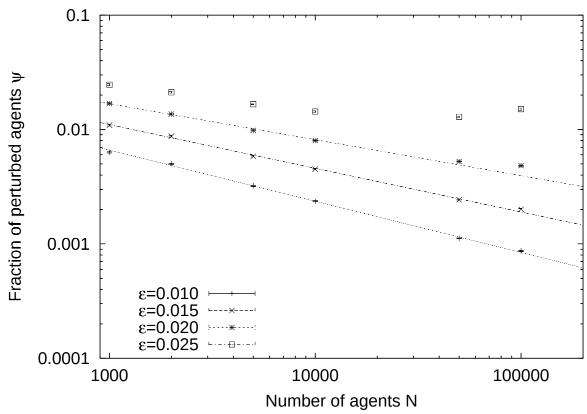

FIG. 3: Dependence of Ψ on the population N for low values of ϵ.

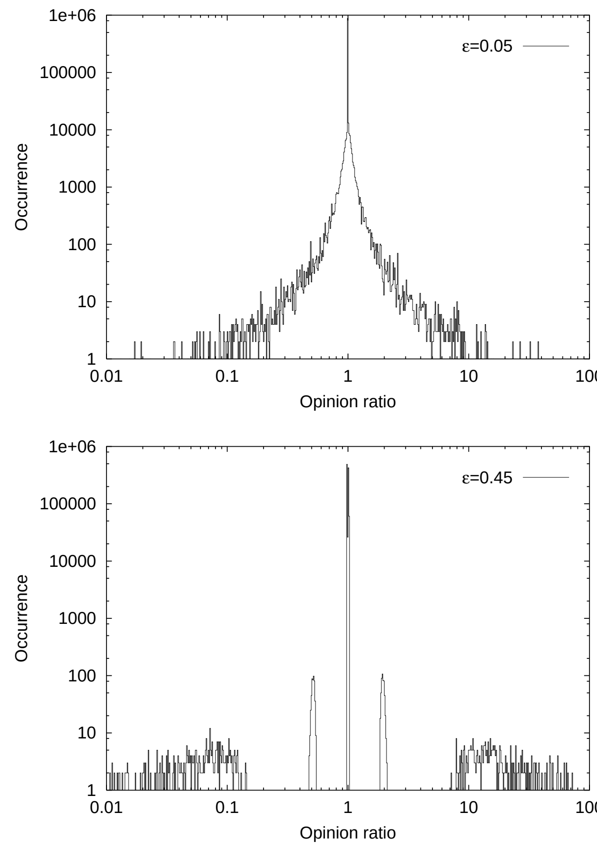

FIG. 4: Histogram of the ratios of the agents' opinions with and without damage. The confidence bound is $\epsilon = 0.05$ (top) and 0.45 (bottom); the population is $N = 1000$.

as well for the opinion dynamics of Deffuant et al. By studying the dependence of this threshold on N, we extrapolated its infinite-N limit $\epsilon_s = 0.500(1)$. We remark that this is just the threshold for complete consensus of our system. So, if we are in the consensus regime and a single agent suddenly changes its mind, this suffices to (slightly) modify the final dominant opinion of the total population.

We tried to check whether this unexpected feature is specific of the particular social topology we have chosen, or whether it is exclusively due to the dynamics. Simulations of a society where agents sit on the sites of a square lattice, with periodic boundary conditions, confirm the result and ϵ_s is again 1/2. We have examined as well a community where each agent has relationships with all others; here we have that ϵ_s is about 0.07, much lower than the consensus threshold 0.21. For this special society there is no DS transition, as each agent effectively interacts with a finite fraction of the system, and it can be proved that $\epsilon_d(N)$ vanishes when $N \to \infty$.

We remind that we have introduced the damage right at the beginning of the evolution. We have also performed some tests to check what happens if we instead perturb the system after some evolution steps. Now the perturbed agent stays in a society where people are mostly divided in groups of close opinions, and such communities will evolve separately from each other. In this way the shocked agent can interact only with a smaller portion of the system, i.e. with a few clusters of agents, and the amount of damage will drop. On the other hand, we find that both ϵ_d and ϵ_s remain the same.

Furthermore we have studied the effective opinion variations of the agents if damage is introduced. For this purpose we divided the opinion of each agent in the perturbed configuration by the corresponding value in the unperturbed configuration. In Fig. 4 we show the distribution of the opinion ratios at the end of the time evolution, for two values of the confidence bound. As we can see, when ϵ is small, the distribution is strongly peaked at the value 1, and the opinions vary continuously though very little in most cases. As we approach the consensus threshold, instead, we notice that the opinion variations are no longer continuously distributed, and other narrow peaks appear, which shows that most values of the ratios are suppressed and discontinuous jumps, corresponding to drastic opinion changes, are allowed.

Let us now check what happens when the initially damaged agent sits on a node with low degree. In Fig. 5 we compare the DS curve obtained in this case with the curve for a perturbed hub. We see that, for any ϵ, Ψ is larger when we damage the hub, as expected. On the other hand we find that both ϵ_d and ϵ_s are the same as before. That relies on the small world effect [16] on scale free networks like ours. In fact, each node can reach any other through a small number of intermediaries. In this way, even if we perturb a loosely connected node, within the first evolution steps the perturbation will have reached quite a few nodes with much higher degrees, which brings us back to the previous case. The damage is larger if we perturb a hub because more agents can be reached at the beginning of the evolution; soon after that, as we said above, clusters of opinions are formed

which do not interact with each other and the perturbation can exclusively spread within the affected clusters. In conclusion, due to the small world effect, Ψ and the fraction $\Phi = 1 - \Psi$ of unperturbed agents are of the same order of magnitude in both situations: if Ψ (Φ) is zero in one case, it will be zero in the other case too.

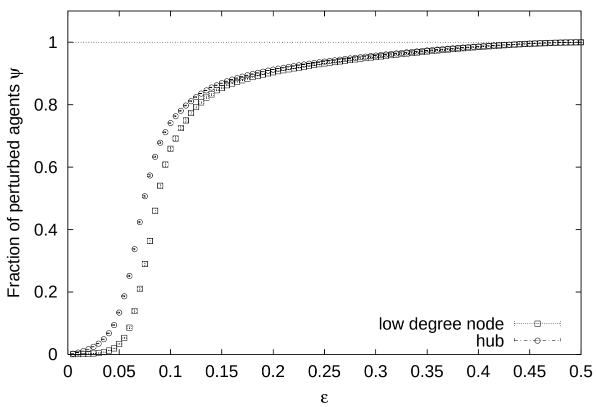

FIG. 5: Comparison of the DS curves corresponding to the initial perturbation of a node with low degree (squares) or a hub (circles). The number of agents is $N = 1000$.

We have studied damage spreading for the Krause-Hegselmann opinion dynamics on Barabási-Albert networks. We distinguish three phases in the confidence bound space, corresponding to zero, partial and total damage, respectively. The existence of a phase where the perturbation affects all agents is new for damage spreading processes and is independent of the social topology, so it only relies on the dynamics. This feature seems unrealistic, but we should consider that our dynamics is very simple [17]. Moreover, we let our system reach in any case its final state, but the evolution time grows with the size of the community and would be very long for a realistic population of voters; normal electoral campaigns last a few months, so we should interrupt the evolution process at an earlier stage and the damage could then be limited. The probability to have large variations of the final agents' opinions induced by the initial perturbation is rather small, but it increases with ϵ and, by approaching the consensus threshold, forbidden bands appear. The amount of the damage depends on the degree of the damaged node but the thresholds for damage spreading and saturation do not, because of the small world effect.

In the future it would be helpful to use other opinion dynamics to check for the consistency of the results. For more realistic analyses of the problem, together with eventual refinements of the existing consensus models and of the social topology, it is important to include as well other factors in the dynamics, like advertising and noise.

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